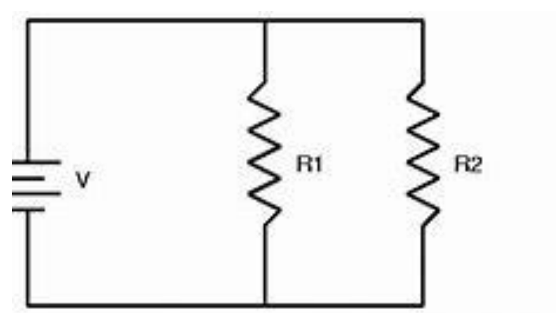
APPLICATIONS OF FRACTIONAL EQUATIONS

❑ INTRODUCTION

❑ ELECTRONICS

The electrical circuit shows a voltage source, along with two resistors wired in **parallel**. The TOTAL resistance, R_T, in the circuit is related to the resistors R_1 and R_2 by the formula



$$\boxed{\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}}$$

EXAMPLE 1: **A parallel circuit contains two resistors, one with a resistance of 8Ω and the other with a resistance of 5Ω. Find R_T, the total resistance in the circuit.**

Resistance can be measured in a unit called the ***ohm***, (named after Georg Ohm) and abbreviated using the Greek capital letter omega, Ω.

Solution: Using the formula we can write

$$\frac{1}{R_T} = \frac{1}{8} + \frac{1}{5}$$

$$\Rightarrow \quad \frac{1}{R_T} = \frac{13}{40}$$

$$\Rightarrow \quad R_T = \frac{40}{13}$$ (take the reciprocal of each side of the equation)

$$\Rightarrow \quad \boxed{R_T \approx 3.08\ \Omega}$$

EXAMPLE 2: **Solve the total resistance formula for R_1.**

Solution: We start, of course, with the given formula:

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$$

The LCD is the product of all three denominators, which is $R_T R_1 R_2$, so we multiply each side of the formula by the LCD, which clears out the fractions:

$$\left[R_T R_1 R_2\right]\left(\frac{1}{R_T}\right) = \left[R_T R_1 R_2\right]\left(\frac{1}{R_1}\right) + \left[R_T R_1 R_2\right]\left(\frac{1}{R_2}\right)$$

Now simplify by cross-canceling:

$$R_1 R_2 = R_T R_2 + R_T R_1$$

$$\Rightarrow \quad R_1 R_2 - R_T R_1 = R_T R_2$$

$$\Rightarrow \quad R_1(R_2 - R_T) = R_T R_2$$

$$\Rightarrow \quad \boxed{R_1 = \frac{R_2 R_T}{R_2 - R_T}}$$

Homework

1. A parallel circuit contains two resistors, one with a resistance of 5Ω and the other with a resistance of 0.2Ω. Find R_T, the total resistance in the circuit.

2. The total resistance in a parallel circuit containing two resistors is $\mathbf{8\Omega}$. If one of the resistors has a resistance of $\mathbf{10\Omega}$, find the resistance of the other resistor.

3. Solve the total resistance formula for R_T.

4. Solve the total resistance formula for R_2.

❑ *MORE MOTION PROBLEMS*

A while back, the key to solving motion problems was to use the formula $rt = d$: The product of the rate (speed) and the time is the distance. We can solve for t by dividing each side of the formula by r, thereby getting a formula for time:

$$\boxed{t = \frac{d}{r}}$$

This formula should make sense. For instance, if you travel a distance of 200 miles at an average rate of 50 miles per hour, your travel time is

$$t = \frac{d}{r} = \frac{200 \text{ miles}}{50 \text{ miles per hour}} = 4 \text{ hours}$$

This formula for *time* is the key to the next two examples.

A note for you science people: What justifies the above calculations where *miles* over *miles per hour* turned into hours? I will try to prove to you that miles divided by miles per hour must turn into hours:

$$\frac{\text{mi}}{\frac{\text{mi}}{\text{hr}}} = \frac{\frac{\text{mi}}{1}}{\frac{\text{mi}}{\text{hr}}} = \underbrace{\frac{\text{mi}}{1} \div \frac{\text{mi}}{\text{hr}} = \frac{\text{mi}}{1} \times \frac{\text{hr}}{\text{mi}}}_{\text{Multiply by the reciprocal}} = \frac{\cancel{\text{mi}}}{1} \times \frac{\text{hr}}{\cancel{\text{mi}}} = \frac{\text{hr}}{1} = \textbf{hr}$$

EXAMPLE 3: **Maria hikes up a 30-mile mountain trail. She then rents a pair of skis and skis back down the trail. Her skiing speed is 9 mph faster than her hiking speed. If Maria spent a total of 7 hours on the trail, what were her hiking speed and her skiing speed?**

Solution: Let's set up a table, with the top row stating our standard motion formula: Rate × Time = Distance. The next two rows are labeled "Hiking up" and "Skiing down."

	Rate ×	Time =	Distance
Hiking up			
Skiing down			

The first thing we can see is that the distance hiking up the mountain and the distance skiing down the mountain are the same, 30 miles. Also, since the speed skiing down the mountain is 9 mph faster (more than) the hiking speed up the mountain, we can let h represent the hiking speed, which implies that $h + 9$ is the skiing speed:

	Rate	× Time	= Distance
Hiking up	h		30
Skiing down	$h + 9$		30

So far, so good. But what about the Time column? The problem says nothing about the time it took to hike up the mountain or the time it took to ski back down to the bottom. Here's where the time formula discussed earlier, $t = \frac{d}{r}$, comes to our rescue. For the "Hiking up" row, using the formula for t and the values of r and d in the table, we get

$$t = \frac{d}{r} = \frac{30}{h},$$

and for the "Skiing down" row we get

$$t = \frac{d}{r} = \frac{30}{h+9}$$

Our table is now completely filled in:

	Rate	× Time	= Distance
Hiking up	h	$\frac{30}{h}$	30
Skiing down	$h + 9$	$\frac{30}{h+9}$	30

Now it's time for an equation. The only given information we haven't used yet is that Maria spent a <u>total of 7 hours</u> on the trail — part of the 7 hours hiking up and the other part skiing down. In other words, the hiking time plus the skiing time equals 7 hours; we have our equation:

$$\frac{30}{h} + \frac{30}{h+9} = 7$$

This is a fractional equation; the LCD is $h(h + 9)$, so we multiply each side of the equation by the LCD:

$$\frac{30}{\cancel{h}}\left[\cancel{h}(h+9)\right] + \frac{30}{\cancel{h+9}}\left[h\cancel{(h+9)}\right] = 7\left[h(h+9)\right]$$

A little cross-canceling simplifies things to

$$30(h + 9) + 30(h) = 7(h^2 + 9h)$$

which distributes out to become the quadratic equation

$$30h + 270 + 30h = 7h^2 + 63h$$

Bringing all the terms to the right to achieve standard quadratic form gives

$$0 = 7h^2 + 3h - 270$$

From here we factor and set each factor to 0:

$$0 = (7h + 45)(h - 6)$$

$$\Rightarrow \quad 7h + 45 = 0 \quad \text{or} \quad h - 6 = 0$$

$$\Rightarrow \quad h = -\frac{45}{7} \quad \text{or} \quad h = 6$$

A negative rate doesn't make sense in this problem, so we'll discard it and keep the value of 6 for h, the hiking speed, making the skiing speed $h + 9 = 6 + 9 = 15$. We're finally done:

Her hiking speed was **6 mph** and her skiing speed was **15 mph**.

<u>EXAMPLE 4:</u> **Jason can go 15 miles upstream in his motorboat in the same time he can go 63 miles downstream. If the speed of his boat is 13 mph in still water, what is the rate of the current?**

<u>Solution:</u> We again create a table, with our distance formula in the top row, and "Upstream" in the second row and "Downstream" in the third row. Let's put some data into our

table right now, noting that the upstream distance is 15 mi while the downstream distance is 63 mi.

	Rate	× Time	= Distance
Upstream (against the current)			15
Downstream (with the current)			63

Next, we analyze the rate (speed) column of the table. First, we see that the problem tells us that Jason's motorboat can go 13 mph in still water, without any current affecting the speed.

Second, notice that the problem asks for the rate of the current. It's time for the variable; we'll let c = the rate of the current.

Third, we need to carefully think about the meaning of the terms "upstream" and "downstream." Going upstream means that the boat is going in the opposite direction of the current, which <u>hinders</u> the boat's normal speed. Thus, the upstream rate is the boat's normal speed <u>minus</u> the rate of the current:

$$\text{upstream rate } = 13 - c$$

Going downstream means that the boat is going in the same direction as the current, which <u>boosts</u> the boat's normal speed.

So the downstream rate is the boat's normal speed <u>plus</u> the rate of the current:

$$\text{downstream rate } = 13 + c$$

We can now fill in the Rate column of the table:

	Rate ×	**Time** =	**Distance**
Upstream (against the current)	$13 - c$		15
Downstream (with the current)	$13 + c$		63

The expressions that go into the Time column are based on the time formula we wrote in the previous example: $t = \dfrac{d}{r}$

	Rate ×	**Time** =	**Distance**
Upstream (against the current)	$13 - c$	$\dfrac{15}{13-c}$	15
Downstream (with the current)	$13 + c$	$\dfrac{63}{13+c}$	63

It's now time to come up with an equation. Notice the phrase "*in the same time*" in the problem. This means that even though Jason traveled different distances at different speeds as he motorboated upstream and downstream, it took the <u>same amount of time</u> for each part of the trip. Hence, our equation is found by setting the two Times equal to each other:

$$\frac{15}{13-c} = \frac{63}{13+c}$$

Multiply each side of the equation by the LCD, $(13 - c)(13 + c)$:

$$\left[(13-c)(13+c)\right]\frac{15}{13-c} = \frac{63}{13+c}\left[(13-c)(13+c)\right]$$

Cross-cancel the common factors:

$$\left[\cancel{(13-c)}(13+c)\right]\frac{15}{\cancel{13-c}} = \frac{63}{\cancel{13+c}}\left[(13-c)\cancel{(13+c)}\right]$$

This leaves

$$15(13 + c) = 63(13 - c)$$

$$\Rightarrow \quad 195 + 15c = 819 - 63c$$

$$\Rightarrow \quad 78c = 624$$

$$\Rightarrow \quad \underline{c = 8}$$

We conclude that

$$\boxed{\text{The rate of the current is 8 mph.}}$$

Homework

5. Phil hikes up a 32-mile mountain trail. He then rents a pair of skis and skis back down the trail. His skiing speed is 8 mph faster than his hiking speed. If Phil spent a total of 6 hours on the trail, what were his hiking speed and his skiing speed?

6. Raya can go 9 miles upstream in her motorboat in the same time she can go 33 miles downstream. If the speed of her boat is 7 mph in still water, what is the rate of the current?

7. Ann hikes up a 30-mile mountain trail. She then rents a pair of skis and skis back down the trail. Her skiing speed is 4 mph faster than her hiking speed. If Ann spent a total of 8 hours on the trail, what were her hiking speed and her skiing speed?

8. Marcus can go 7 miles upstream in his motorboat in the same time he can go 147 miles downstream. If the speed of his boat is 11 mph in still water, what is the rate of the current?

Review Problems

9. Millie hikes up a 30-mile mountain trail. She then rents a pair of skis and skis back down the trail. Her skiing speed is 10 mph faster than her hiking speed. If Millie spent a total of 8 hours on the trail, what were her hiking speed and her skiing speed?

10. Joseph can go 35 miles upstream in his motorboat in the same time he can go 175 miles downstream. If the speed of his boat is 15 mph in still water, what is the rate of the current?

Solutions

1. $R_T \approx 0.192\ \Omega$

2. $\mathbf{40}\,\Omega$

3. $R_T = \dfrac{R_1 R_2}{R_1 + R_2}$

4. $R_2 = \dfrac{R_1 R_T}{R_1 - R_T}$

5. 8 mph and 16 mph

6. 4 mph

7. 6 mph and 10 mph

8. 10 mph

9. 5 mph and 15 mph

10. 10 mph

"It is the Law that any difficulties that can come to you at any time, no matter what they are, must be exactly what you need most at the moment, to enable you to take the next step forward by overcoming them. The only real misfortune, the only real tragedy, comes when we suffer without learning the lesson."

– *Emmet Fox*